Promotion of cooperation by payoff noise in a 2×2 game

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A series of numerical simulations of a 2×2 symmetric game on a network examined whether payoff matrix noise promotes cooperation, as reported initially by Perc [New J. Phys. **8**, 22 (2006)]. Agents have no memory (they offer cooperation, *C*, or defection, *D*). We assume that the network is time invariable. The effect of payoff matrix noise (PMN) is measured by a simulated payoff difference between a normal network game and a network game with PMN. The effect of PMN appears only when a local strategy adaptation is implemented (for example, a network game with imitation dynamics). The influence of PMN becomes more significant with a larger stochastic deviation, and less significant in a larger degree network. One reason for PMN's effectiveness is the local strategy adaptation mechanism, which helps both the preservation and fixation of *C* agents, and not that the payoff matrix noise makes a dilemma game into a Trivial (dilemma-free) game.

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I. INTRODUCTION

The emergence of cooperation in overcoming a dilemma has been explained by several theories. In terms of evolutionary game theory, Nowak [1] classified five mechanisms that make cooperation (*C*) evolve instead of defection (*D*): Kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. Network reciprocity relies on two effects. The first is limiting the number of game opponents (depressing anonymity), which leads to a rise in mutual cooperation, and the second is a local adaptation mechanism, in which an agent copies a strategy from a neighbor linked by a network. These explain how *C* agents survive in a network game of Prisoner's Dilemma (PD), e.g., even though it requires agents to use only the simplest strategy—either *C* or *D*.

In the past few years, many studies have dealt with network reciprocity. Masuda and Aihara [2] investigated how cooperation emerges, with the spatial Prisoner's Dilemma (PD) played on a class of networks ranging from a regular lattice to random networks through small-world (SW) topology. They concluded that SW is the optimum structure for enabling cooperators to thrive through cluster formation. Hauert and Szabo [3] reported a series of similar experiments with a remark that the advantages that elicit cooperation through networks work, but are rather limited. Tomochi [4] insisted that SW networks, which get increasingly disordered (e.g., the shortcut probability increases), decrease the cooperation fraction. In relation to this point, Ren *et al.* [5] suggested the presence of shortcuts in SW as randomness in the network. They demonstrated that a system with such randomness often evolves into a cooperative state; however, there is an optimal amount of randomness, which induces the highest level of cooperation. Further, they pointed out that the mechanism by which randomness promotes cooperation resembles a resonance. Tomassini *et al.* [6] examined SW networks for Chicken games, insisting that cooperation is sometimes inhibited and sometimes enhanced, depending on the update rules (whether they are asynchronous or synchronous, replicator dynamic update or proportional update, etc.) and the game structure.

Another important heterogeneous network is the scale-free (SF) network. Several studies (e.g., Gomez-Gardenes [7]; Santos and Pacheco [8]; Santos *et al.* [9]; Tang *et al.* [10]) reported that cooperation is enhanced in SF networks, since they form a single cluster containing the most connected players (hubs). However, Hu *et al.* [11] reported that, for a SF network, the assumption of whether the maximum degree hub agent is *C* or *D* at the beginning of a simulation episode crucially affects the dynamics that follow—almost determining whether the equilibrium would be a cooperative or defective phase. Lopez-Pintado [12] examined Stag Hunt games on a SF network, and found that SFs do not always support this type of contagion.

Beyond those studies that deal with a dilemma game on a fixed network, Ohtsuki and his colleagues obtained a simple principle by a series of analytical approaches with supplementary simulation studies [13–15]. Assuming a Donor-Recipient game (a special type of PD having $P=0$, $R=b-c$, $S=-c$, and $T=b$, where P, R, S, and T are game-intrinsic elements, as explained later in the Model section), they describe a surprisingly simple rule that is a good approximation for all graphs, including cycles, spatial lattices, random regular graphs, random graphs, and SF networks. Natural selection favors cooperation if the benefit of the altruistic act b divided by the cost c exceeds the average number of neighbors k (average degree), which means $b/c>k$. If a game satisfies $b/c>k$, which is very analogous to Hamilton's rule [1] ($b/c>1/r$, where r is relatedness), cooperation can evolve. Their finding shows that network reciprocity is mostly determined by the average degree, irrespective of the type of network assumed, if a Donor-Recipient game is assumed.

Those earlier studies are based on a framework where agents are initially allocated in a fixed network. Zimmermann *et al.* [16] demonstrated a variant of a coevolution system in a networking game. Their model can consider simultaneous evolution of networks and strategy. Applying this model to several PDs, they observed a stable cooperation phase when a cooperative hub agent (which they called a

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TABLE I. Payoff matrix for a 2×2 game.

		Opponent	
		Cooperation	Defect
Focal	Cooperation (C)	$R+\upsilon_R$	$S+\upsilon_S$
	Defect (D)	$T+\upsilon_T$	$P+\upsilon_P$

C leader) emerged, resulting in C chains. Another significant study, by Pacheco *et al.* [17,18], deals with both networking and strategy adaptations. They adopted two parameters: A time scale for strategy updating and another for network updating. When the former is smaller than the latter, this would be an evolutionary game in a fixed network. Assuming a complete graph as an initial network, this case can be analytically dealt with using the replicator dynamics of a 2×2 game. They are analytically formulated for situations where the network updating scale is much less than that for strategy updating, which can also be evaluated by other replicator dynamics using the 2×2 game matrix that is revised from the original. Their findings are not directly applicable to situations when both time scales seem to be close. These must be solved by a numerical approach, such as that used by Zimmermann *et al.*

In evolutionary games with network reciprocity, the effect of noise (or randomness, in other words) becomes important, since the noise may enhance phase change from a defector dominant state to a cooperator dominant state, which occurs in a resonancelike fashion. The network noise effect as in Ren's statement [5] is a resonance phenomenon. Szabo *et al.* [19] and Vukov *et al.* [20] studied PD on several network topologies, focusing on the effects of payoffs (dilemma strength) and noise on the maintenance of cooperation. The "noise" in their terminology means randomness in the process of strategy adaptation, and is usually called "temperature" K in the Fermi function. They found that a noise effect is influenced by the underlying network structure, especially in networks containing a "one-site overlapping triangle."

Perc and his colleagues presented comprehensive reports on this kind of resonance phenomenon. Initially, Perc [21] presented a report which dealt with the payoff matrix noise (PMN). PMN is defined as the deviations from each P, R, S, and T (in the present paper, those are defined by υ_P, υ_R, υ_S, and υ_T as shown in Table I). He observed a considerable amount of resonance from PMN, of which standard deviation σ is subjected to temporally and spatially white additive Gaussian noise. Later, Perc and Marhl [22] examined numerical calculations with the two- and three-strategy pair approximated analytical approach for PD assuming $k=4$. To compare these, they demonstrated two resonance effects: One from the noise defined by temperature K and the other from PMN. Perc [23] also focused on superposed resonance effects from PMN and network noise. For the PMN, in addition to the Gaussian noise, he examined Levy distribution [24] and chaos variables [25].

Limiting this discussion to the effects of a stochastically deviated payoff matrix, we must cite the work of Eriksson and Lindgren (e.g., [26,27]) as a precursor. They examined how cooperation emerges, and what complex strategies (not simply C or D) evolve when repeated games with stochastic observable payoffs are imposed on agents.

Guan *et al.* [28] investigated what happens when an agent observes amplified payoffs for neighbors in a copy strategy event in a networked PD game. They found a robust C-support effect, although it seems trivial. Because an amplified payoff they defined indicates a positive bias from the original payoff structure, it always increases C agents in a PD.

In Perc's studies [21,24,25], he also reported that PMN can promote cooperation in PD on a 2D lattice network with degree $k=4$. In their model, an agent plays PD games with four von Neumann neighbors, and copies the strategy (either C or D) from one of them, according to the Fermi function defined by the payoff difference between himself and each neighbor. In their model, the dilemma can be resolved locally (a PD can be transformed into a dilemma-free game, i.e., Trivial game) by implementing a larger PMN (namely, larger σ), even though the noise average remains zero. In this case, the efficient C-support phenomenon reported by Perc can be attributed to the game class transformation that we discuss later. This raises a substantial question about how PMN supports cooperation, if the game class (in other words, dilemma strength) is preserved. The present study focuses on this question across a range of network topologies and game structure, which might help in further understanding the resonance effect promoted by the PMN, originally observed by Perc. The resonance effect of PMN might be interesting not only to physicists but also to social scientists, because the effect might be linked to the global behavior of stock markets under a nonuniform information environment.

II. MODEL

Let us presume a 2×2 game on a time-constant network. The number of agents in the society is N. Each agent has k links, which means the degree of an agent is maintained at k. Table I indicates respective payoffs, determined by strategies of both focal and opponent agents (either C or D). Additive components to game intrinsic elements (P, R, S, and T); υ_P, υ_R, υ_S, and υ_T, are stochastic variables, obeying a Gaussian distribution, for which average (μ) and standard deviation (σ) are defined as 0 and $a\times(R-P)$, respectively. Parameter a indicates an amplification. The set υ_P, υ_R, υ_S, and υ_T indicates a Gaussian noise. An agent plays k different games, since a set of additive components is defined in each network link.

According to Tanimoto and Sagara [29], every game defined in a 2×2 game space can be expressed by

$$P = x_0 - 0.5 r_1 \cos\left(\frac{\pi}{4}\right), \tag{1}$$

$$R = x_0 + 0.5 r_1 \cos\left(\frac{\pi}{4}\right), \tag{2}$$

$$S = x_0 + r_2 \cos\left(\frac{\pi}{4} + \theta\right), \tag{3}$$

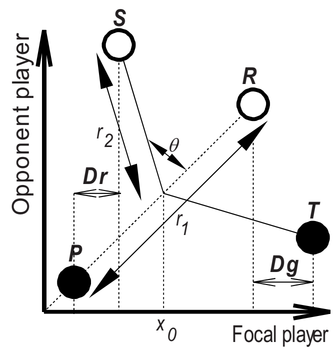

FIG. 1. Schematic expression of a 2×2 game. This example indicates a PD. X axis and Y axis indicate both players' payoffs. Open circle indicates that focal player adopts a cooperation strategy (C), while closed circle indicates that focal player defects (D).

$$T = x_0 + r_2 \sin\left(\frac{\pi}{4} + \theta\right). \quad (4)$$

Figure 1 shows a schematic expression of Eq. (1). As shown in Fig. 1, the expression of Eq. (1) provides a straightforward understanding of the geometric relation between a possible solution set and the game structure of an arbitrary 2×2 game. Since x_0 is independent of the relative relationships among payoffs, a single set of two parameters, $r=r_2/r_1$ and θ (rad), is sufficient to see the entire 2×2 game world, as shown in Fig. 2. One marvelous feature is that this allows us to draw the well-known typical dilemma games, such as PD, Chicken, Stag Hunt (SH), Leader, and Hero, and illustrate the regions in which they occur. Tanimoto and Sagara [29] also show that dilemma in a 2×2 game can be quantified with two game structural parameters—$D_g=T-R$ and $D_r=P-S$. D_g indicates the static-dilemma intensity—the

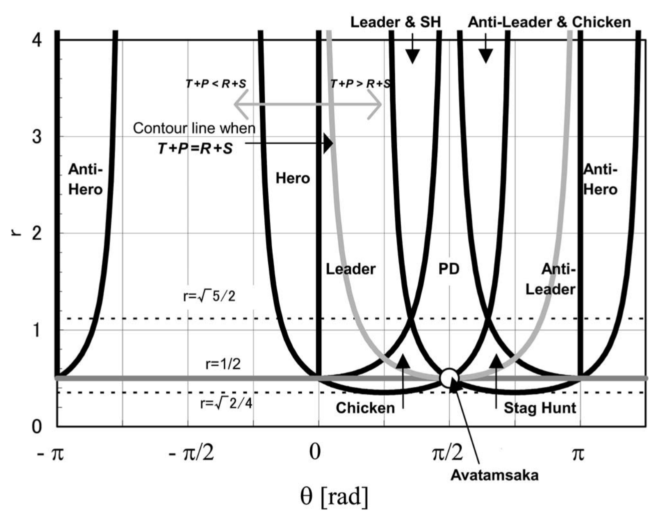

FIG. 2. Scene of 2×2 game world. According to Tanimoto and Sagara [29], any 2×2 game can be parametrized by two game-structural parameters, r and θ. Any game classes, including dilemma games such as PD, Chicken, SH, and even a trivial game can be drawn schematically. One specific game, called Avatamsaka, devised by Akiyama and Aruka [33], stands on a marginal point between dilemma and trivial games.

inclination of two equal players to exploit each other. They call this situation the gamble-intending dilemma (GID). Moreover, D_r indicates the static-dilemma intensity of equal players trying never to be exploited. This is called a risk-aversion dilemma (RAD). They also showed that the actual game dilemma can be explained by the game's structure (static elements—D_g and D_r) and dynamic influences.

It might seem that the description in Eq. (1), based on polar coordinates, is more particular than those presented by early studies (e.g., [30,31]). However, this method is useful in understanding the following points. As explained in later parts, we divide the PMN into two types; x_0 fluctuation ($v_P=v_R=v_S=v_T$) and game-class fluctuation (independent v_P, v_R, v_S, and v_T). From Fig. 1, Table I, and Eq. (1), we can say that adding the same noise to game intrinsic elements (namely, giving $v_P=v_R=v_S=v_T$) preserves a relative relation among P, R, S, and T, which can be paraphrased by saying that x_0 fluctuates around P, R, S, and T, but never changes the game class. For example, if a PD satisfies $v_P=v_R=v_S=v_T$, the PMN never changes the game class of the PD, since the x_0 fluctuation does not affect D_g and D_r. On the other hand, for game-class fluctuation, if v_P, v_R, v_S, and v_T are completely independent, the relative relation among P, R, S, and T can be changed, which means the PMN undergoes a game class transformation.

When $g(\mu,\sigma)$ expresses a random number, obeying a Gaussian distribution defined by μ and σ, the x_0 fluctuation assumes the following:

$$v_P = v_R = v_S = v_T = g_i(0,\sigma),$$

and the game class fluctuation assumes

$$v_P = g_i(0,\sigma), \quad v_R = g_j(0,\sigma), \quad v_S = g_k(0,\sigma), \quad v_T = g_l(0,\sigma).$$

An agent in the present model has no memory. Each agent deterministically copies the strategy (either C or D) from one of his k neighbors (agents connected by his links) who obtained the largest payoff in the previous time step. This is called imitation dynamics. These strategy adaptation processes operate synchronously.

We investigate several networks: 1D ring (1D-R), 2D lattice (2D-L), and regular random network (RRG), by varying $k=4$, 8, and 16.

III. NUMERICAL EXPERIMENT

The assumed experimental parameters are $N=2500$, $r_1=1.272$, and $x_0=0$. Quantitative assumptions of both game structural parameters r_1 and x_0 are not substantial, since r and θ together are sufficient to visualize the entire 2×2 game world, as explained in the preceding section. The initial distribution of C, imposed at the beginning state of every simulation episode, is assumed to be 0.5. The results we discuss below were confirmed robust for those parameters. We vary the game structure $-\frac{3}{4}\pi \leqslant \theta \leqslant \frac{5}{4}\pi$ and $0 \leqslant r \leqslant 5$ in Eq. (1) (or Fig. 1). The contours shown are drawn by ensemble averages of five equilibrium trials (quasi-steady state of the dynamics) for respective game structures (we confirmed that a five-ensemble average seems acceptable to ob-

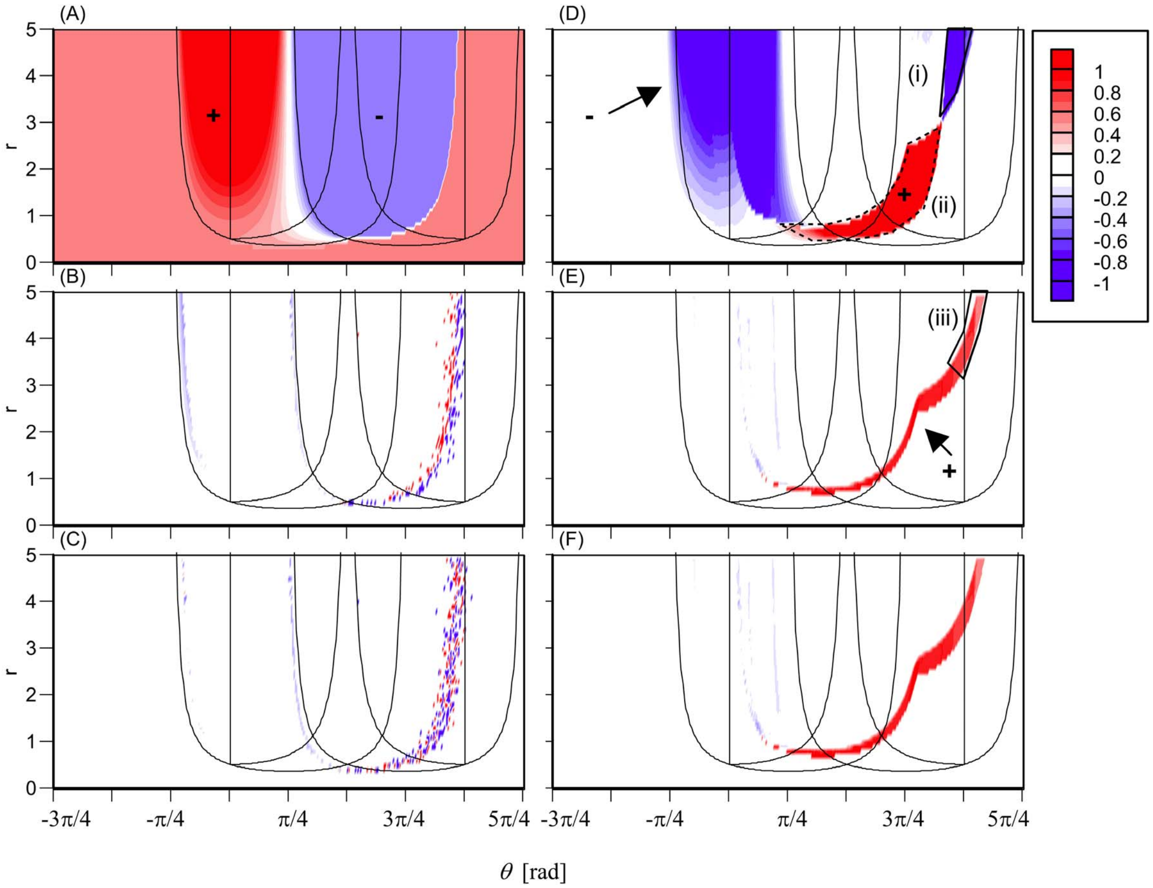

FIG. 3. (Color online) Simulation results of the payoff. (A) Analytic solution (AS); (B) payoff differences between GA with the PMN by the x_0 fluctuation and AS; (C) GA with the PMN by the game-class fluctuation and AS; (D) 1D $k=4$ network and AS; (E) 1D $k=4$ networks with and without the PMN by the x_0 fluctuation; (F) 1D $k=4$ networks with and without the PMN by the game-class fluctuation. + and − indicate positive and negative differences, respectively.

serve the general tendency discussed in the following text).

One simulation episode runs until the time variations of social-averaged cooperation fraction and payoff can be regarded sufficiently small—after 2000 time steps, which seems an asymptotic equilibrium.

Parameter a in a Gaussian noise is basically assumed to be 0.1.

IV. RESULTS

PMN effect on a network

It might be interesting to confirm whether the PMN resonance effect works only on a network structure, or whether it also works in a well-mixed population, implying a global strategy adaptation system.

Figure 3(A) indicates the payoff of an analytic solution (AS) based on replicator dynamics, normalized by $(R-P)$. Because $x_0=0$, the values of R and P in this graph are 0.5 and −0.5, respectively. The AS from replicator dynamics assumes infinite N and a well-mixed population.

Figures 3(B) and 3(C) indicate payoff differences between using a genetic algorithm (GA) for global adaptation and using AS. In Fig. 3(B), $v_P=v_R=v_S=v_T$ is assumed (x_0 fluctuation case), which means there is no game class fluctuation but just an x_0 fluctuation, since the relative relation among P, R, S, and T is preserved, as explained earlier. Whereas, in Fig. 3(C), v_P, v_R, v_S, and v_T are independent Gaussian noises (game class fluctuation), in which the game class may be transformed locally, for instance from a PD to Trivial (dilemma-free game). Those two payoff differences are also normalized by $(R-P)$ (all the following results are the same). Since we cannot observe any significant differences with AS, this implies that the PMN never works to encourage cooperation under a global strategy adaptation system, such as GA.

Figure 3(D) shows a payoff difference between 1D-R, $k=4$, and AS, which implies the effectiveness of a network in the absence of other C-support mechanisms. We can see obvious network effectiveness in area (ii) spanning from Chicken to PD, SH, and Anti-Leader (confirm in Fig. 1). Nowak [1] called this network reciprocity. In this particular game area, because of the relatively small dilemma strength expressed by D_g and D_r, the network effect can lead a game trail to a cooperative state (all cooperators or coexistence cooperators and defectors at least), by forming C clusters. However, a negative effectiveness is spreading in Leader and Hero. In those two particular game areas satisfying

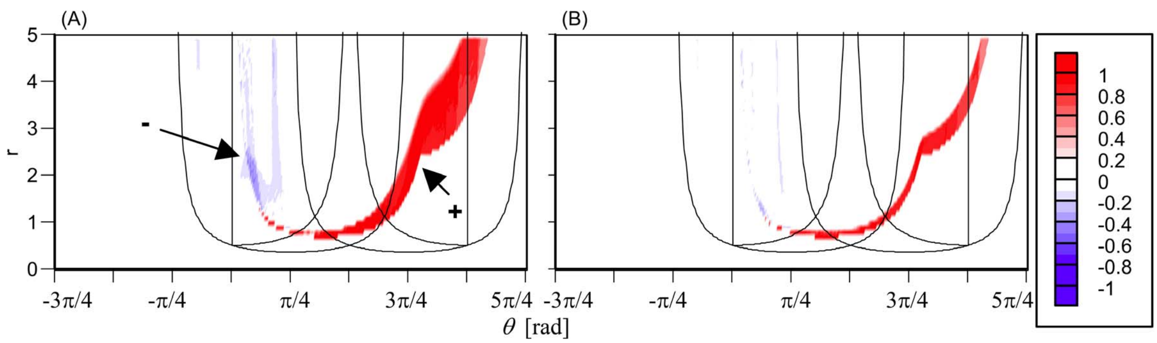

FIG. 4. (Color online) Simulation results of the payoff indicating payoff differences between with and without the PMN by the x_0 fluctuation on 1D $k=4$ networks. (A) assumes Gaussian distribution with $\sigma=0.3\times(R-P)$. (B) assumes uniform distribution with $\sigma=0.1\times(R-P)$. + and − indicate positive and negative differences, respectively.

$S+T>2R$, R-reciprocity—where both agents offer C—is less effective in obtaining a higher payoff than ST-reciprocity—where focal and opponent agents offer C and D alternately [32]. Thus the network reciprocity does not support ST-reciprocity in Leader and Hero games.

Area (i) (in Anti-Leader spanning to Anti-Hero) indicates a negative network effect from the AS. In this area, the game structure satisfies $P>S$ and $P>T$. In the case of an AS with no mechanisms to support cooperation, Anti-Leader and Anti-Hero lead to an all-defection state (every agent obtains P), if the initial C distribution is 0.5 [see Fig. 3(A)]. But in area (i), despite the instinct game structure that encourages obtaining P, mutual defections become impossible due to the network effect. In this area, very few small C clusters can remain in a D majority. A central agent in the C cluster obtains R (the largest payoff), but neighboring D agents on the border of the C-D front impose S (the least payoff if in Anti-Leader) on the C agents. These C agents cannot change their strategy from C to D, because the central C agent compels them to retain C by copying. This is why certain fractions of S and T occur in the area, which leads to a social-average payoff less than the AS.

One might notice that the area of positive effectiveness (or even the negative effectiveness area in Anti-Leader) does not coincide with the game borders. This is not unusual, because the graph indicates the payoff difference, e.g., in the case of PDs having $\theta=0$ (these are Donor and Recipient games, a special case of the PD game class) that network reciprocity works effectively only to support cooperation within a smaller or moderate dilemma strength, where the area of positive effectiveness [area (ii)] is observed. With increasing r (increasing dilemma strength), network reciprocity becomes insufficient to support cooperation, of which equilibrium is led to all defectors state that is the same as the case for AS [shown by empty space in Fig. 3(D)].

Figure 3(E) shows a payoff difference between considering and neglecting the x_0 fluctuation for a 1D-R, $k=4$ network, which precisely indicates the effectiveness of PMN in encouraging cooperation on the basis of the network structure through x_0 fluctuation. Figure 3(F) shows the payoff difference between considering and neglecting the game-class fluctuation for a 1D-R, $k=4$ network. These payoff differences are not with AS [as in Figs. 3(B) and 3(C)] but with the payoff only considering network reciprocity (1D-R, $k=4$ in those two cases) to determine the effectiveness of PMN either by x_0 fluctuation or game-class fluctuation (all the following results are expressed in same manner). Figure 3(D) indicates the impact of topology, and Figs. 3(E) and 3(F) show the further impact of noise.

One thing to note here is the obvious effectiveness of PMN in the relatively narrow area spanning from Chicken to PD, SH, and Anti-Leader. This, added to what we observed in Figs. 3(B) and 3(C), implies that PMN only works to encourage cooperation under a local strategy adaptation system, such as games on a network. In addition, we also should note that the effectiveness of PNM over network reciprocity seems only moderate. Noting that area (iii) in Fig. 3(E) is almost consistent with area (i) of Fig. 3(D), we assume that the negative network effect is diluted in area (iii) due to the effect of PMN. Moreover, we notice that the other positive area, besides area (iii) in Fig. 3(E), is on the boundary of area (ii), Fig. 3(D). We infer from this that the PMN effect is significant in bolstering the positive network effect and in alleviating the negative network effect in Anti-Leader and Anti-Leader region.

Another important point to note is that the PMN effect is sufficient due to a local-payoff fluctuation, since we observe no significant difference between Figs. 3(E) and 3(F). This seems interesting because it implies that the resonance effect required to support cooperation does not come from game class alterations brought on by PMN.

Hence we only see the x_0 fluctuation cases in the following text.

Influence of noise quality and quantity

Figure 4(A) shows the payoff difference between considering and neglecting the x_0 fluctuation for a 1D-R, $k=4$ network in which the standard deviation (SD) of the Gaussian distribution σ for the x_0 fluctuation is assumed to be $0.3\times(R-P)$ [$a=0.3$ is assumed instead of 0.1, as in Fig. 3(E)]. The result of comparison of Fig. 3(E) to this proves that the effectiveness of PMN increases with an increase in matrix noise σ.

Figure 4(B) shows the payoff difference between considering and neglecting the x_0 fluctuation for a 1D-R, $k=4$ net-

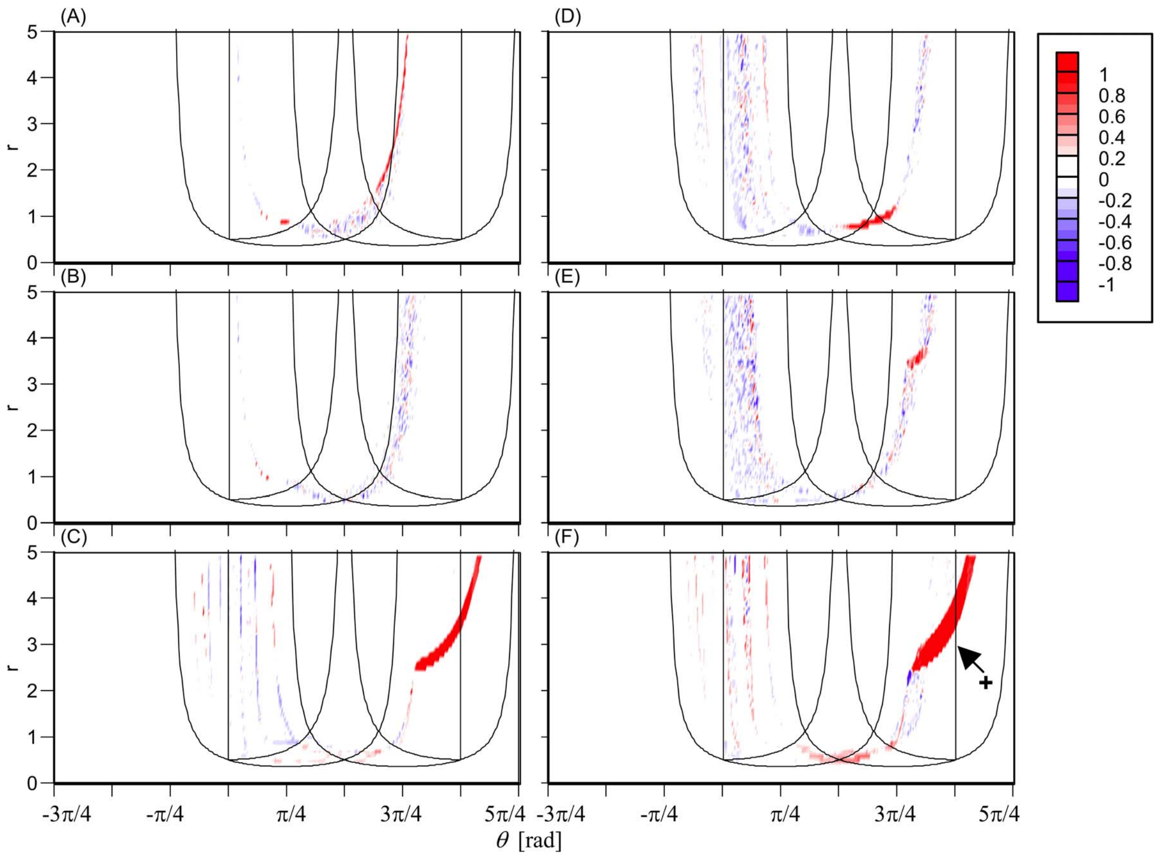

FIG. 5. (Color online) Simulation results of the payoff indicating payoff differences between with and without the PMN by the x_0 fluctuation assuming Gaussian distribution with $\sigma=0.1\times(R-P)$. (A) 1D $k=8$, (B) 1D $k=16$ networks, (C) 2D $k=4$ networks (von Neumann lattice), (D) 2D $k=8$ networks (Moore lattice), (E) 2D $k=16$ networks, and (F) 2D $k=4$ Random Regular networks. + indicates positive difference.

work in which the x_0 fluctuation assumes a uniform distribution with $\sigma=0.1\times(R-P)$. Compared with Fig. 3(E), there is no significant difference, which implies that PMN's effectiveness is insensitive to the assumed probabilistic distribution when the same SD is imposed.

Influence of networks on PMN

It might be interesting to investigate how different network topologies affect PMN's effectiveness. In the following results, all the cases assume a Gaussian distribution $\sigma=0.1\times(R-P)$ when considering the x_0 fluctuation. Figure 5(A) indicates a payoff difference between considering and neglecting the x_0 fluctuation for a 1D-R, $k=8$ network. Figures 5(B)–5(F) are for 1D-R, $k=16$; 2D-L, $k=4$ (von Neumann lattice); 2D-L, $k=8$ (Moore lattice); 2D-L, $k=16$; and RRG, $k=4$, respectively.

Both 2D-L, $k=4$ and RRG, $k=4$ cases show the effectiveness of PMN in the Anti-Leader area, while other cases only show a few positive areas. Observing those results, we notice that the effectiveness of PMN in Fig. 3(E) fades rapidly as the degree of the network increases.

Ohtsuki *et al.* [15] analytically proved that the advantage of network reciprocity disappears in a large-degree network, since that situation approximates a well-mixed population. The average network path length (L) and the cluster coefficient (C) are well-known parameters (as well as the average degree k) for evaluating network characteristics. In particular, L seems appropriate for evaluating how slowly a local strategy spreads into the population, which indicates the advantage of a local strategy adaptation over a global strategy adaptation. In general, a smaller L indicates that the situation approximates a well-mixed population, while a structural network has a larger L. Table II shows both L and C for each network. As shown in Fig. 5, PMN's effectiveness for RRG, $k=4$ is almost the same as for 2D-L, $k=4$, and is much greater than that for 1D-R, $k=16$, even though the L for RRG, $k=4$ is smaller than either 2D-L, $k=4$ or 1D-R, $k=16$. This implies that PMN's effectiveness is not dependent on the average path length, but depends strongly on the degree of a network. As mentioned before, network reciprocity relies on two effects: Limiting the number of game opponents (depressing anonymity) and the local strategy adaptation mechanism. In terms of depressing anonymity, a regular graph is superior to a RRG, when they have a same degree. Comparing Figs. 5(C) and 5(F), we observe that depressing anonymity seems ineffective for PMN's effectiveness. Therefore, from a qualitative point of view, the local strategy adaptation mechanism seems more meaningful for the PMN's effectiveness on a network than depressing anonymity.

TABLE II. Cluster coefficient and average path length for each network.

Network structure	Degree, K	Cluster coefficient, C	Average length of path, L
1D	4	4.9×10^{-1}	312.0
	8	6.4×10^{-1}	156.6
	16	6.7×10^{-1}	78.6
2D	4	0	25.0
	8	4.3×10^{-1}	16.7
	16	4.0×10^{-1}	7.0
Random regular graph	4	4.0×10^{-4}	6.4
	8	2.3×10^{-3}	4.0
	16	9.4×10^{-3}	2.9

V. DISCUSSION

In a series of Perc's studies, the supporting C effect might come from game-class fluctuations, where the PD transforms to Trivial (dilemma-free game), because of the PMN [24]. The present report, however, proved that there is no significant difference between the game-class fluctuation and x_0 fluctuation under the assumed SD range. Even if a game-class alteration does not occur locally, a C-supporting effect can be encouraged; an agent copies the C strategy from a neighboring C agent who obtains a high payoff because of the payoff matrix noise, even though this particular neighbor might be exploited by D agents. Note that the C-support effect of PMN arises only from x_0 fluctuation. This implies that the substance of the PMN does not come from game-class alteration occurring locally, but rather from the local-strategy adaptation system, where imitation dynamics provides possibilities for preserving C strategy in some local patches.

However, under the assumptions we imposed for the PMN, network reciprocity seems more important than PMN in obtaining a high payoff or high cooperation fraction. As Nowak indicated [1], network reciprocity is more effective for a smaller k (cooperation can evolve in the PD defined by $P=0$, $R=b-c$, $S=-c$, and $T=b$ if $1/k>c/b$ is satisfied). Considering both Nowak's statement and the present result, we expect a larger C-support effect in a dilemma game intending R reciprocity, if the PMN is combined with a smaller degree network.

VI. CONCLUSIONS

We investigated the effect of payoff matrix noise (PMN) by varying the network and 2×2 game structures. The PMN, originally reported by Perc, states that a probabilistic fluctuation, having a zero average and a certain SD, applied to game-intrinsic elements encourages a higher C fraction in a dilemma game with a network structure.

(1) The PMN never works under a global-strategy adaptation system, such as GA, for a well-mixed population.

(2) The PMN works when a local-strategy adaptation system is assumed. Hence this effect can be applicable to a dilemma game in a network structure, where an agent copies C or D from his neighbors for the next time step. However, the PMN appears in a limited fashion, within a narrow dilemma-game region, spanning from Chicken to PD, SH, and Anti-Leader, where the network reciprocity can solve the dilemma.

(3) The PMN works sufficiently only because of the x_0 fluctuations that do not require independent noises—allowing, for example, game-class alteration from a PD to Trivial (dilemma-free game).

(4) The PMN is significantly influenced by both the deviation of the fluctuation and the degree of the network.

[1] M. A. Nowak, Science **314**, 1560 (2006).
[2] N. Masuda and K. Aihara, Phys. Lett. A **313**, 55 (2003).
[3] C. Hauert and G. Szabo, Am. J. Phys. **73**, 405 (2005).
[4] M. Tomochi, Soc. Networks **26**, 309 (2004).
[5] J. Ren, W. X. Wang, and F. Qi, Phys. Rev. E **75**, 045101(R) (2007).
[6] M. Tomassini, L. Luthi, and M. Giacobini, Phys. Rev. E **73**, 016132 (2006).
[7] J. Gomez-Gardenes, M. Campillo, L. M. Floria, and T. Moreno, Phys. Rev. Lett. **98**, 108103 (2007).
[8] F. Santos and J. Pacheco, Phys. Rev. Lett. **95**, 098104 (2005).
[9] F. Santos, J. Rodrigues, and J. Pacheco, Proc. R. Soc. London, Ser. B **273**, 51 (2006).
[10] C. L. Tang, W. X. Wang, X. We, and B. H. Wang, Eur. Phys. J. B **53**, 411 (2006).
[11] M. B. Hu *et al.*, Eur. Phys. J. B **53**, 273 (2006).
[12] D. Lopez-Pintado, Int. J. Game Theory **34**, 371 (2006).
[13] H. Otsuki and M. A. Nowak, Proc. R. Soc. London, Ser. B **273**, 2249 (2006).
[14] H. Ohtsuki and M. A. Nowak, J. Theor. Biol. **243**, 86 (2006).
[15] H. Ohtsuki, C. Hauert, E. Lieberman, and M. A. Nowak, Nature (London) **441**, 502 (2006).
[16] M. G. Zimmermann and V. M. Eguiluz, Phys. Rev. E **72**, 056118 (2005).
[17] J. M. Pacheco, A. Traulsen, and M. A. Nowak, J. Theor. Biol. **243**, 437 (2006).
[18] J. M. Pacheco, A. Traulsen, and M. A. Nowak, Phys. Rev. Lett. **97**, 258103 (2006).
[19] G. Szabo, J. Vukov, and A. Szolnoki, Phys. Rev. E **72**, 047107 (2005).
[20] J. Vukov, G. Szabo, and A. Szolnoki, Phys. Rev. E **73**, 067103 (2006).
[21] M. Perc, New J. Phys. **8**, 22 (2006).
[22] M. Perc and M. Marhl, New J. Phys. **8**, 142 (2006).
[23] M. Perc, New J. Phys. **8**, 183 (2006).
[24] M. Perc, Phys. Rev. E **75**, 022101 (2007).
[25] M. Perc, Europhys. Lett. **75**, 841 (2006).

[26] A. Eriksson and K. Lindgren, Evolution of strategies in repeated stochastic games with full information of the payoff matrix, Proceedings of GECCO, 2001, pp. 853–859.
[27] A. Eriksson and K. Lindgren, Artif. Life **VIII**, 394 (2002).
[28] J. Y. Guan, Z. X. Wu, X. J. Huang, and Y. H. Wang, Europhys. Lett. **76**, 1214 (2006).
[29] J. Tanimoto and H. Sagara, BioSystems (to be published).
[30] R. Harris, Behav. Sci. **14**, 138 (1969).
[31] C. H. Hauert, Int. J. Bifurcation Chaos Appl. Sci. Eng. **12**, 1531 (2002).
[32] J. Tanimoto and H. Sagara, BioSystems (to be published).
[33] E. Akiyama and Y. Aruka, 2004, *The Effect of Agents Memory on Evolutionary Phenomena—the Avatamsaka Game and Four Types* 2×2 *Dilemma Games*, Proc. of 9th Workshop on Economics and Heterogeneous Interacting Agents, CD-ROM.